Algebraic cluster model calculations for vibrational to gamma-unstable shape phase transition in odd-A nuclei

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Abstract
The Algebraic Cluster Model(ACM) is an interacting boson model that gives the relative motion of the cluster congurations in which all vibrational and rotational degrees of freedom are present from the outset. We schemed a solvable extended transitional Hamiltonian based on ane SU(1;1) Lie algebra within the framework for two-, three- and four- body algebraic cluster models that explains both regions O(4) U(3), O(7) U(6) and O(10)-U(9), respectively. We offer that this method can be used to study of k + x nucleon structures with k = 2, 3, 4 and x = 1, 2, . . .

Keywords: Quantum Phase Transition, Interacting Boson Fermion Model, SU(1,1) algebra, Algebraic Cluster Model(ACM)

Introduction
Algebraic models are advantageous in the many-body and in few-body systems. In algebraic models energy eigenvalues and eigenvectors are obtained by diagonalizing a finite-dimensional matrix, rather than by solving a set of coupled differential equations in coordinate space. The aim of this paper is to discuss the quantum phase transitions in the algebraic cluster models for the two-, three- and four- body cluster, to transition description in U(3)-O(4), U(6)-O(7) and U(9) - O(10). This model can be solved by using an infinite dimensional algebraic technique in the IBFM framework and was applied to the k + x nucleon structures consisting of k - particles and x nucleons. The expectation value of boson number operator and behavior of the overlap of the ground-state wave function within the control parameters of this evaluated Hamiltonian are presented.

Theoretical Framework
We consider two dynamical symmetries of the ACM Hamiltonian for the n-body problem which are related to the group lattice

$$U(3n-3) \supset \left\{ \begin{matrix} U(3n-2) \\ O(3n-3) \end{matrix} \right\} \supset O(3n-2) \qquad (1)$$

which are called the U(3n- 2) and SO(3n- 23) limits of the ACM, respectively. A geometric analysis shows that the U(3n-2) limit corresponds for large N to the (an)harmonic oscillator in 3(n-1) dimensions and the SO(3n-3) limit to the deformed oscillator in 3(n-1) dimensions[1,2].

The method for diagonalization of the Hamiltonian in the transitional region is not as easy as in either of the limits, especially when the dimension of the configuration space is relatively large. To avoid these problems, an algebraic Bethe ansatz method within the framework of an infinite dimensional SU(1; 1) Lie algebra has been proposed by Pan et al [3].

The boson algebraic structure will be taken to two-, three- and four cluster systems with $j^f = \frac{1}{2}$ as:

$$U^B(4) \otimes U^F(2) \supset \left\{ \begin{matrix} U^B(3) \\ O^B(4) \end{matrix} \right\} \otimes SU^F(2) \supset O^B(3) \otimes SU^F(2) \supset Spin^{BF}(3)$$

$$U^B(7) \otimes U^F(2) \supset \left\{ \begin{matrix} U^B(6) \\ O^B(7) \end{matrix} \right\} \otimes SU^F(2) \supset O^B(6) \otimes SU^F(2) \supset O^B(3) \otimes SU^F(2) \supset Spin^{BF}(3)$$

$$U^B(10) \otimes U^F(2) \supset \left\{ \begin{matrix} U^B(9) \\ O^B(10) \end{matrix} \right\} \otimes SU^F(2) \supset O^B(9) \otimes SU^F(2) \supset O^B(3) \otimes SU^F(2) \supset Spin^{BF}(3)$$

The following Hamiltonian for description of negative and positive states in transitional region for two-cluster systems is prepared

$$H = gS_0^+ S_0^- + \alpha S_1^0 + \beta \hat{C}_2(\mathrm{O}^B(3)) + \gamma \hat{C}_2(\mathrm{Spin}^{BF}(3)) \qquad (2)$$

By employing the generators of Algebra SU (1; 1) and Casimir operators of subalgebras , the following Hamiltonian for transitional region for three-cluster is suggested as

$$H = gS_0^+ S_0^- + \alpha S_1^0 + \beta \hat{C}_2(\mathrm{O}^B(6)) + \gamma \hat{C}_2(O^B(3)) + \eta \hat{C}_2(\mathrm{Spin}^{BF}(3)) \qquad (3)$$

In a four-cluster model, the Hamiltonian for the transitional region can be considered as

$$H = gS_0^+ S_0^- + \alpha S_1^0 + \beta \hat{C}_2(\mathrm{O}^B(9)) + \gamma \hat{C}_2(O^B(3)) + \eta \hat{C}_2(\mathrm{Spin}^{BF}(3))$$

(4)

For evaluating the eigenvalues of the suggested Hamiltonians, the eigenstates are considered as

$$\left|k; v_s v_b n_\Delta, \mathrm{L}\, JM\right\rangle = \Theta S^+_{x_1} S^+_{x_2} S^+_{x_3} \ldots S^+_{x_k} \left|lw\right\rangle^{BF} \quad (5)$$

The eigenvalues of Hamiltonians Eqs. (2), (3) and (4) can then be expressed;

$$E^{(k)} = h^{(k)} + \alpha\Lambda_1^0 + \beta L(L+1) + \gamma J(J+1) + \eta T(T+1) \quad (6)$$

$$E^{(k)} = h^{(k)} + \alpha\Lambda_1^0 + \beta v(v+4) + \gamma L(L+1) + \eta J(J+1) \quad (7)$$

$$E^{(k)} = h^{(k)} + \alpha\Lambda_1^0 + \beta v(v+7) + \gamma L(L+1) + \eta J(J+1) \quad (8)$$

$$h^{(k)} = \sum_{i=1}^{k} \frac{\alpha}{x_i} \quad (9)$$

Results and discussion

The quantal order parameter that we mention here are the expectation values of the boson number operators. The expectation values of n_b are the significant objectives of phase transition. So, we calculated these values to show the treatment of phase transition. In order to calculate the expectation values of the b-boson number operator, we have to select the suitable roots. Given the proper amount of roots, we have calculated $\langle n_b \rangle$ for two, three and four - clusters in odd-A nuclei.

Fig.1 shows the expectation values of the b-boson number operator for the lowest states odd-A nuclei as a function of control parameter for N = 10 bosons. The sudden change in these quantities show the phase transition. Figures show that the expectation values of the number of vector-bosons remain approximately constant for a limit and only begin to change rapidly for the other limit.

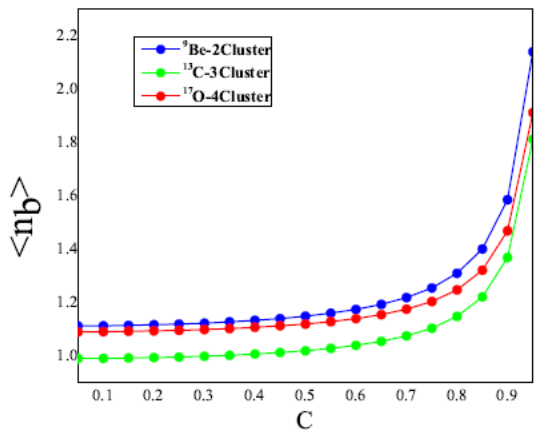

FIG. 1. The expectation values of the vector-boson number operator for the lowest states as a function of control parameter C for N=10.

It has been shown that the overlap of the ground-state wave function with that in the dynamical symmetries may also serve as a signature of the phase transition [4]. We have calculated the overlap of the ground state wave functions of the Hamiltonian (2). The obtained results are illustrated in Fig.2.

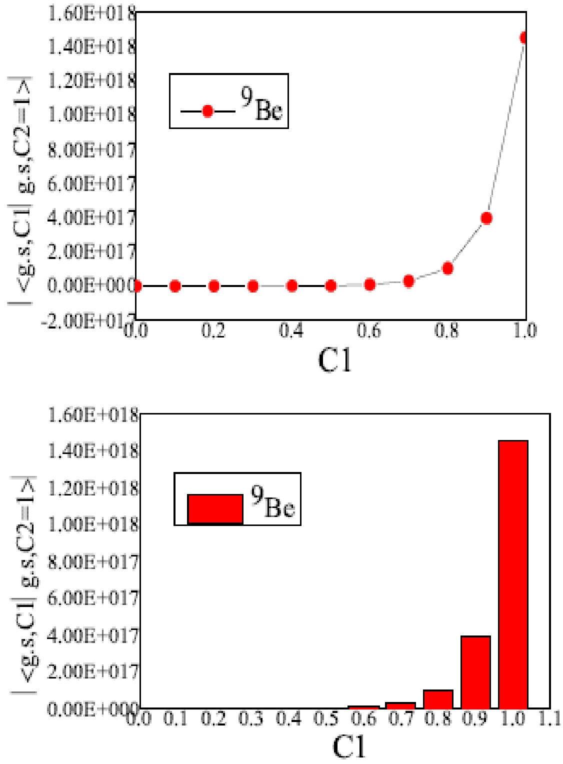

FIG. 2: The Calculated variation behavior of the overlap of the ground-state wave function as a function of control parameter C for N=10 .

Conclusions

In this paper, we have studied the phase transitions of the algebraic cluster models. solvable extended transitional Hamiltonian which is based on SU(1; 1) algebra is proposed to investigation of quantum phase transition between the spherical and the deformed phases. solvable extended transitional Hamiltonian which is based on SU(1; 1) algebra is proposed to pave the way for of quantum phase transition between the spherical and the deformed phases. for investigating odd-A and odd-odd nuclei. So, the clustering survives the addition of one and two particles. we have presented here a analysis of quantum phase transitions in a system of N bosons and one fermion.

References

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