



Studying the existence of quasi-bound state in $K\bar{K}NN$ system

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Abstract

In present work, the Faddeev AGS equations were solved in momentum representation for $K\bar{K}NN$ four-body system with quantum numbers $J = 0$ and $I = 0$. The Faddeev calculations are based on the quasi-particle method and the method of the energy dependent pole expansion was used to obtain the separable representation for the integral kernels in the three- and four-body equations. Different types of $\bar{K}N - \pi\Sigma$ potentials based on phenomenological and chiral SU(3) approaches were used. As a remarkable result of this investigation, it was found that the four-body $K\bar{K}NN$ system is bound with a binding energy of $B_{K\bar{K}NN} = 38-83$ MeV and width of $\Gamma_{K\bar{K}NN} = 50-114$ MeV.

Keywords: Kaonic systems, $\bar{K}N$ interaction, $\Lambda(1405)$ resonance, Faddeev method

Introduction

The $\bar{K}N$ interaction, which is affected by $\Lambda(1405)$ resonance, plays an important role in the exotic systems, including the antikaon particle [1,2]. Thus, to study the kaonic systems, it is necessary to know the $\bar{K}N$ interaction. The first prediction of a quasi-bound state in kaonic nuclear systems was made in [2,3], showing that these systems could be strongly bound. For the past two decades, many theoretical and experimental searches were performed, focusing on the three- and four-body kaonic systems (especially K^-pp system) [4-15].

The purpose of the present paper is to explore the binding energy and width of four-body $K\bar{K}NN$ system including kaon and antikaon particles. The problem can be solved using methods developed within four-body theories [16-18].

Three and four-body calculations

The quantum numbers of the $K\bar{K}NN$ system are $J = 0$ and $I = 0$, in actual calculations, when we include isospin and spin degrees of freedom the number of configurations is equal to eighteen, corresponding to different possible two-quasi-particle partitions. In the case of $K\bar{K}NN$ system, we have one pair of identical nucleons. Therefore, we will have five different subsystems.

$$\begin{aligned} \sigma = 1: & K + (\bar{K}NN); & \sigma = 2: & \bar{K} + (KNN) \\ \sigma = 3: & N + (K\bar{K}N); & \sigma = 4: & (K\bar{K}) + (NN) \\ \sigma = 5: & (KN) + (\bar{K}N) \end{aligned} \quad (1)$$

Since, in Faddeev method the dynamics of all particles can be fully included, in present calculations, we used the Faddeev AGS method. The whole dynamics of $K\bar{K}NN$ four-body system is described in terms of the transition amplitudes $A_{\sigma(i)\rho(j);\mu\nu}^{IiIj}$ which connect all quasi-two-body channels. Antisymmetrization of nucleons to be made within each channel. The Faddeev equations for kaonic systems under consideration can be expressed by [19]

$$\begin{aligned} & A_{\sigma(i)\rho(j);\mu\nu}^{IiIj} \\ &= R_{\sigma(i)\rho(j);\mu\nu}^{IiIj} + \sum_{\tau;kl;\lambda\kappa} \sum_{I_k I_l} R_{\sigma(i)\tau(k);\mu\lambda}^{IiIj} \vartheta_{kl;\lambda\kappa}^{\tau;I_k I_l} A_{\sigma(l)\rho(j);\kappa\nu}^{IiIj} \end{aligned} \quad (2)$$

Here, the operators $A_{\sigma(i)\rho(j);\mu\nu}^{IiIj}$ are the four-body transition amplitudes, which describe the dynamics of the four-body $K\bar{K}NN$ system and the $\vartheta_{kl;\lambda\kappa}^{\tau;I_k I_l}$ functions are the effective propagators. To define the spectator particle or interacting particles in each two- and three-body subsystem, we used the i, j and k indices and the isospin of the interacting particles are defined by I_i . The indices μ, ν are used for defining which term of the separable expansion of the subamplitudes is used. The operators $R_{\sigma(i)\rho(j);\mu\nu}^{IiIj}$ are driving terms, which describe the effective particle-exchange potential realized by the exchanged particle between the quasi-particles in channels σ and ρ .

Results and discussion

Before we proceed to represent the obtained results, we will have a survey on the two-body interactions. The two-body interactions are the central input to our few-body calculations. We used separable potentials in momentum representation in the form

$$V_I^{\alpha\beta} = g_I^\alpha \lambda_I^{\alpha\beta} g_I^\beta \quad (3)$$

where g_I^α is the form factor of the interacting two-body with isospin I and $\lambda_I^{\alpha\beta}$ is the strength parameter of the interaction. The interactions are further labeled by α values to take the $\bar{K}N - \pi\Sigma$ and $\bar{K}K - \pi\pi - \pi\eta$ couplings into account. To describe the $\bar{K}N$ interaction, which plays a crucial role in the present calculations, we considered two different phenomenological [20] and one chiral potential [21]. Depending on a pole structure of the $\Lambda(1405)$, we refer phenomenological potentials as



“ $SIDD^1$ ” and “ $SIDD^2$ ” potential. For nucleon-nucleon interaction is the one-term PEST potential from Ref. [22], which is a separable approximation of the Paris model of NN interaction.

We constructed our own potentials for the coupled-channel $\bar{K}K - \pi\pi$ and $\bar{K}K - \pi\eta$ interactions. To define the parameters of the potential, we used the mass and width of the f_0 and a_0 (mass 985 MeV/c² and the width 60 MeV) resonances and the $\bar{K}K$ scattering length [23]. For KN interaction with isospin $I = 0,1$, the range parameters of the potentials were set to 3.9 fm⁻¹ and the strength parameters are adjusted to reproduce the KN scattering length [24-26].

Starting from Faddeev AGS equations (2) and using different versions of the $\bar{K}N - \pi\Sigma$ potentials, the binding energy and width of the $K\bar{K}NN$ quasi-bound state were evaluated. In Table (1), the pole position of the quasi-bound states in the $K\bar{K}NN$ system are presented. The energy-dependent potentials provide a weaker $\bar{K}N$ attraction for lower energies. Therefore, one expects that, the quasi-bound states resulting from the energy-dependent potential happen to be shallower. The comparison of the obtained results for the chiral $\bar{K}N$ interaction with the calculated binding energies for phenomenological $\bar{K}N$ interaction shows that energy-independent potentials produce much deeper bound state.

To solve the four-body Faddeev equations, we used the quasi-particle method [16-18]. The key point of the quasi-particle method is the separable representation of the off-shell scattering amplitudes in two- and three-body subsystems. To reduce the four-body Faddeev equations to a set of single-variable integral equations, one can employ different methods [17,27]. One can do the reduction procedure numerically by making use of the so-called HSE method proposed by Narodetsky [17] and also by using the energy-dependent pole expansion method which developed by Sofianos *et al.*, [27]. Using the EDPE method, we found the separable expressions for the [3+1] and [2+2] subsystems.

Tables

Table 1. Dependence of the pole position (in MeV) of the $K\bar{K}NN$ system to the $\bar{K}N$ model of interaction.

$\bar{K}N$ model	$SIDD^1$	$SIDD^2$	chiral
$K\bar{K}NN$ pole	2786-i57	2816-i25	2831-i30

To take the coupling between $\bar{K}N$ and $\pi\Sigma$ channels and the coupling between $\bar{K}K$, $\pi\pi$ and $\pi\eta$ directly into account, the formalism of Faddeev equations in (1) should be extended to include the particle channels [28]. Thus, all operators should have particle indices for each

state in addition to the Faddeev indices. In the present calculations, the $\pi\Sigma$ channel of the $\bar{K}N - \pi\Sigma$ and $\pi\pi$ and $\pi\eta$ channel of the $\bar{K}K - \pi\pi - \pi\eta$ system have included indirectly and one-channel Faddeev AGS equations are solved for the $K\bar{K}NN$ system. We approximated the full coupled-channel interaction by constructing the so-called exact optical potential [28].

Conclusion

In summary, we extracted the binding energy and width of the four-body $K\bar{K}NN$ system by using the Faddeev-type calculations. To investigate the dependence of the resulting binding energy and width on the $\bar{K}N$ models of interaction, different versions of $\bar{K}N$ potentials having the one- or two-pole structure of $\Lambda(1405)$ resonance, were used. Our calculations show that the $K\bar{K}NN$ system is bound for all models of interaction. The obtained binding energy is 38-83 MeV and the extracted width is about 50-114 MeV.

References

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