The effect of nuclear deformation on thermodynamic properties of ^{96}Mo with inclusion of thermal fluctuations

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Abstract
In this work, we have studied the effect of nuclear deformation on thermodynamic properties of ^{96}Mo. We have used the BCS average value model to consider thermal fluctuations. In this model, thermodynamic quantities are calculated using the mean value of the gap parameter. The results show that nuclear deformation has a significant effect on nuclear level density, as it is found experimentally.

Keywords: Nuclear deformation, BCS average value model, Nuclear level density, Heat capacity.

Introduction
BCS theory was first proposed in 1957 by Bardeen, Cooper, and Schrieffer to justify the behavior of superconductors [1]. This model describes superconductivity in infinite systems of electrons very well. In superconductors, electrons close to Fermi level interact to form a pair known as the Cooper pair.
In nuclei, similar superconducting behavior occurs, which is caused by the pairing interaction between nucleons. The pairing strength is determined by a parameter called the gap parameter, Δ. In the case of nuclei, with limited size and limited number of particles, the effect of thermal fluctuations is not negligible and it is especially important in a phenomenon called the pairing phase transition [2].
The phase transition in superconductors are quite sudden, as a result, the heat capacity suffers discontinuity at critical temperature where the gap parameter becomes zero. The same phenomenon is observed in the nuclear system, with the difference that the phase transition in the nuclei has an S-shaped behavior, which refers to the thermal phase transition in nuclei.

Theory
The BCS superconductivity model uses the most probable value of the gap parameter to calculate the thermodynamic quantities of systems [1]. However, due to the small size of nucleus and the small number of nucleons, there is a fundamental deviation between the most probable value of the gap parameter and its average value. In the BCS average value model, BCS relationships are optimized using the isothermal probability distribution function [3]. In this method, using the isothermal probability distribution function, the mean value of the gap parameter in the BCS model ($\bar{\Delta}$) is calculated and this value is used to determine the thermodynamic quantities of the system. The probability of a system being in a certain state with a certain value of the gap parameter is proportional to e^{Ω_Δ}. In this model, the average value of the gap parameter is obtained from the following equation [3]:

$$\bar{\Delta}_{\beta,\lambda} = \frac{\int_0^\infty e^{\Omega_{\Delta,\beta,\lambda}}\,\Delta\, d\Delta}{\int_0^\infty e^{\Omega_{\Delta,\beta,\lambda}}\, d\Delta} \tag{1}$$

The grand partition function in this model is as follows

$$\Omega = -\beta\sum_{k>0}(\varepsilon_k - \lambda - E_k) + 2\sum_{k>0}\ln(1+e^{-\beta E_k}) - \beta\frac{\bar{\Delta}^2}{G} \tag{2}$$

In the above relation, ε_k is the single particle energy of particles, G is the strength of the interaction, λ is the chemical potential, β is the inverse of the nuclear temperature, $\bar{\Delta}$ is the mean value of the gap parameter, and E_k is the quasi-particle energy without the interaction.
So the thermal quantities such as particle number, excitation energy, entropy, heat capacity and nuclear level density can be calculated [3]

$$\begin{aligned}
N &= \frac{\partial\Omega}{\partial\alpha}\\
E &= -\frac{\partial\Omega}{\partial\beta}\\
S &= \Omega - \alpha N + \beta E\\
C_v &= -\beta\frac{\partial S}{\partial\beta}\\
\rho &= \frac{e^S}{(2\pi)^2}\frac{1}{\sqrt{D}}
\end{aligned} \tag{3}$$

where D is a 3 × 3 determinant containing second-order derivatives of the grand partition function at the saddle point.

Results and discussion

To calculate the energy of the single-particle levels, we use the Nilsson model [4]. Since ^{96}Mo is a deformed nucleus ($\beta_2 = 0.17$), we perform our calculations for two different deformations ($\beta_2 = 0.17$ and $\beta_2 = 0$). First, we calculate the neutron and proton gap parameters versus temperature, shown in figures 1 and 2, respectively. Then, using the gap parameter, other

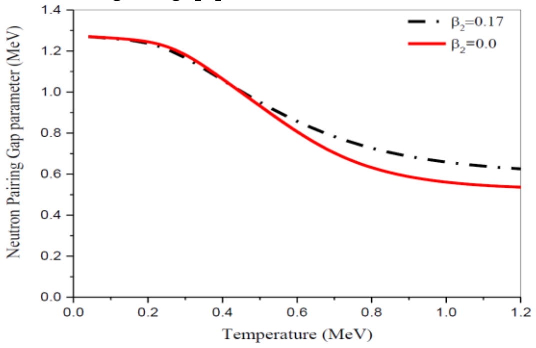

Figure1. Temperature dependence of gap parameter in ^{96}Mo nucleus at two deformations for neutron system.

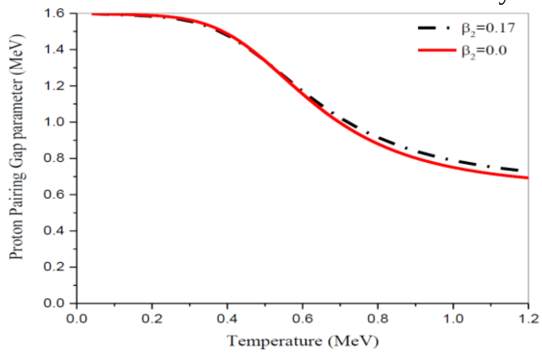

Figure 2. Temperature dependence of gap parameter in ^{96}Mo nucleus at two deformations for proton system.

thermodynamic quantities can be calculated. The calculated heat capacity and nuclear level density at two different deformations are shown in figures 3 and 4, respectively, as well as the experimental results.

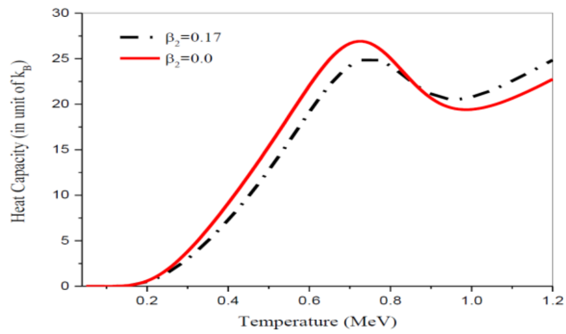

Figure 3. Heat capacity versus temperature in ^{96}Mo nucleus at two deformations.

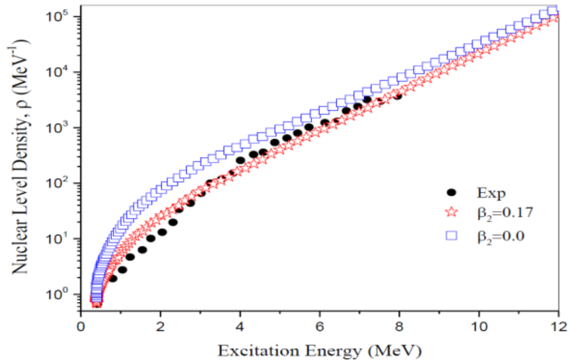

Figure 4. Nuclear level density versus excitation energy in ^{96}Mo nucleus at two deformations. Experimental data were taken from [5].

Conclusions

It can be seen from figures (1) amd (2) that the gap parameter does not vanish with increasing temperature but, rather reaches an almost constant value. For $\beta_2 = 0.0$, the decrease of gap parameter is slighly more rapid at high temperatures. Figure (3) shows the S-shape heat capacity at two deformations which is interpreted as a phase transition from the pairing phase to the normal phase, as it is confirmed experimentally [5]. The calculated nuclear level density versus excitation energy at two different deformations are plotted in figure (4) as well as the experimental results. As it is seen, the experimental results are consistent with the calculated level density at $\beta_2 = 0.17$.

In summary, inclusion of both thermal fluctuations and deformation (β_2=0.17) lead to a significant improvement of the calculated level density compared with the experimental results.

Acknowledgments

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