# Investigation of the use of Momentum and Galerkin weighting functions in High-order Nodal Expansion Method in order to solve the Neutron Diffusion Equation

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## Abstract
Today in nuclear reactor calculations, researchers are looking for methods that, in addition to the acceptable accuracy, have optimal calculation cost. One of the ways to increase the accuracy of calculations without reducing the node size in the nodal expansion method is the use of suitable weighting functions. In this study, after discretization of the neutron diffusion equation and adjoint with high-order nodal expansion method in two dimensions and two energy groups, calculations with Momentum and Galerkin weighting functions for rectangular geometry (BIBLIS-2D) and hexagonal geometry (IAEA-2D) reactors are performed.
**Keywords: Simulator, Adjoint Calculation, Diffusion Equation, Rectangular Geometry, HACNEM.**

## Introduction
One of the basic needs in designing, simulating, and studying nuclear reactors is neutronic analysis. There are various numerical methods such as finite difference, finite volume, finite element, and nodal for spatial discretization of the neutron diffusion equation. Each of these methods can be used according to the desired geometry, the type,and size of the meshes, the required accuracy, and the desired computational time[1-6].
In order to have tools with optimal computational cost, a method that uses large nodes about the size of the fuel assemblies must be used, which shows the importance of using nodal expansion method (NEM) in this type of calculation[3-7].
The main challenge of the nodal expansion method is its relatively low accuracy. In this paper, using the High-order nodal expansion method and nodes with the size of fuel assemblies, the accuracy of the calculations increases significantly. The method is: using different momentum and Galerkin weight functions in the average current nodal expansion method without reducing the size of the nodes and investigating the accuracy of Adjoint and Neutron Diffusion Equation both locally (relative power distribution) and in general (effective multiplication factor).

## Diffusion Equation Solution by NEM
The steady-state multi-group neutron diffusion equation is given in Eq.1. By discretizing Eq.1 according to Fig.1 for Rectangular geometry and performing integration and mathematical simplifications, finally, three equations of balance, coupling, and high order for this geometry are obtained by Eqs 2 to 4, respectively. Also, these steps for hexagonal geometry are performed according to Fig.2, and the equations of balance, coupling and, high order are obtained as Eqs. 5 to 7. It should be noted that the parameters in the equations are available in the references and will be given in the full article.

$$-\nabla . D_g \nabla\ \phi_g(r) + \Sigma_{r,g}\phi_g(r) = \frac{\chi_g}{k_{eff}}\sum_{g'=1}^{G}\nu\Sigma_{f,g'}\phi_{g'}(r) + \sum_{\substack{g'=1\\g'\neq g}}^{G}\Sigma_{s,g'g}\phi_{g'}(r),\ g=1,2,...,G \quad (1)$$

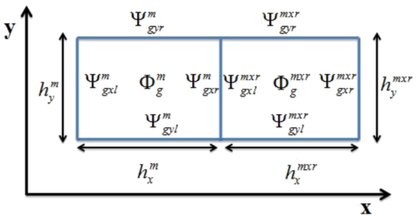


Figure 1. The nodal coordinate system for rectangular geometry.

$$\left[\sum_{u=x,y} 2\frac{A^m_{gu}}{h^m_u} + \Sigma^m_{rg}\right]\Phi^m_g = \sum_{\substack{g'=1\\g'\neq g}}^{G}\Sigma^m_{g'g}\Phi^m_{g'} + \frac{\chi_g}{K_{eff}}\sum_{g'=1}^{G}\nu\Sigma^m_{fg'}\Phi^m_{g'} + \sum_{u=x,y}\frac{1}{h^m_u}[(1-B^m_{gu}-C^m_{gu})(j^{-m}_{gul}+j^{-m}_{gur})-2E^m_{gu}d^m_{gu2}] \quad (2)$$

$$\begin{bmatrix} j^{+m}_{gul} \\ j^{+m}_{gur} \end{bmatrix} = \begin{bmatrix} A^m_{gu} & B^m_{gu} & C^m_{gu} & -D^m_{gu} & E^m_{gu} \\ A^m_{gu} & C^m_{gu} & B^m_{gu} & D^m_{gu} & E^m_{gu} \end{bmatrix} \begin{bmatrix} \Phi^m_g \\ j^{-m}_{gul} \\ j^{-m}_{gur} \\ d^m_{gu1} \\ d^m_{gu2} \end{bmatrix} \quad (3)$$

$$\left\{\frac{D^m_g}{h^{m^2}_u} + A_k\Sigma^m_{rg}\right\}d^m_{guk} = \sum_{\substack{g'=1\\g'\neq g}}^{G}\Sigma^m_{g'g}\left\{A_k d^m_{g'uk} - B_k e^m_{g'uk}\right\} + \frac{\chi_g}{k_{eff}}\sum_{g'=1}^{G}\upsilon\Sigma^m_{fg'}\left\{A_k d^m_{g'uk} - B_k e^m_{g'uk}\right\} + B_k\Sigma^m_{rg}e^m_{guk} + B_k L^m_{guk} \quad (4)$$

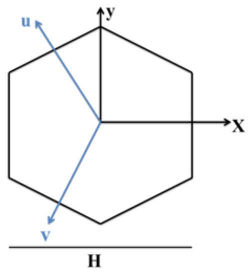


Figure 2. The nodal coordinate system for hexagonal geometry.

$$\left[\frac{4}{H}C_{g5}^{m}+\Sigma_{rg}^{m}\right]\Phi_{g}^{m}=\sum_{\substack{g'=1\\g'\neq g}}^{G}\Sigma_{sg'g}^{m}\Phi_{g'}^{m}+\frac{\chi_g}{K_{eff}}\sum_{g'=1}^{G}\nu\Sigma_{fg'}^{m}\Phi_{g'}^{m}+ \tag{5}$$

$$\sum_{\substack{s=r,l\\w=x,u,v}}\frac{2}{3h}(1-C_{g1}^{m}-C_{g2}^{m}-2C_{g3}^{m}-2C_{g4}^{m})\,j_{gws}^{-m}$$

$$\begin{bmatrix} j_{gxr}^{+m}\\ j_{gxl}^{+m}\\ j_{gur}^{+m}\\ j_{gul}^{+m}\\ j_{gvr}^{+m}\\ j_{gvl}^{+m}\end{bmatrix}=\begin{bmatrix} C_{g1}^{m} & C_{g2}^{m} & C_{g4}^{m} & C_{g3}^{m} & C_{g4}^{m} & C_{g3}^{m} & C_{g5}^{m} & C_{g6}^{m} & C_{g7}^{m} & C_{g7}^{m}\\ C_{g2}^{m} & C_{g1}^{m} & C_{g3}^{m} & C_{g4}^{m} & C_{g3}^{m} & C_{g4}^{m} & C_{g5}^{m} & -C_{g6}^{m} & -C_{g7}^{m} & -C_{g7}^{m}\\ C_{g4}^{m} & C_{g3}^{m} & C_{g1}^{m} & C_{g2}^{m} & C_{g4}^{m} & C_{g3}^{m} & C_{g5}^{m} & C_{g7}^{m} & C_{g6}^{m} & C_{g7}^{m}\\ C_{g3}^{m} & C_{g4}^{m} & C_{g2}^{m} & C_{g1}^{m} & C_{g3}^{m} & C_{g4}^{m} & C_{g5}^{m} & -C_{g7}^{m} & -C_{g6}^{m} & -C_{g7}^{m}\\ C_{g4}^{m} & C_{g3}^{m} & C_{g4}^{m} & C_{g3}^{m} & C_{g1}^{m} & C_{g2}^{m} & C_{g5}^{m} & C_{g7}^{m} & C_{g7}^{m} & C_{g6}^{m}\\ C_{g3}^{m} & C_{g4}^{m} & C_{g3}^{m} & C_{g4}^{m} & C_{g2}^{m} & C_{g1}^{m} & C_{g5}^{m} & -C_{g7}^{m} & -C_{g7}^{m} & -C_{g6}^{m}\end{bmatrix}\begin{bmatrix} j_{gxr}^{-m}\\ j_{gxl}^{-m}\\ j_{gur}^{-m}\\ j_{gul}^{-m}\\ j_{gvr}^{-m}\\ j_{gvl}^{-m}\\ \Phi_{g}^{m}\\ d_{gx}^{m}\\ d_{gu}^{m}\\ d_{gv}^{m}\end{bmatrix} \tag{6}$$

$$\begin{bmatrix}\alpha_g^m & \beta_g^m & \beta_g^m\\ \beta_g^m & \alpha_g^m & \beta_g^m\\ \beta_g^m & \beta_g^m & \alpha_g^m\end{bmatrix}\begin{bmatrix} d_{gx}\\ d_{gu}\\ d_{gv}\end{bmatrix}=\begin{bmatrix} Q_{gx}^m\\ Q_{gu}^m\\ Q_{gv}^m\end{bmatrix} \tag{7}$$

## *Results and discussion*

After calculations with Momentum and Galerkin weighting functions for rectangular geometry (BIBLIS-2D) and hexagonal geometry (IAEA-2D), the results of thermal neutron flux with Momentum weighting functions for recangular and Galerkin weighting functions hexagonal geometry (as a part of results in this extended abstract) are shown in Fig.3. Also the relative power (RP), error of relative rower (ERP), and effective multiplication factors($K_{eff}$) are given in Table1. It should be noted that the execution time of the codes is related to the size of the nodes and has a weak dependency on the different weighting functions.

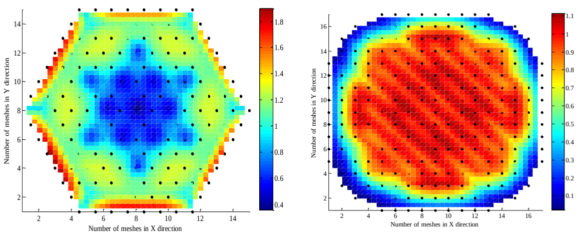

Figure 3. Thermal neutron flux distribution in IAEA-2D (left) and BIBLIS-2D (right) with Galerkin and Momentum weighting functions, respectively.

Table 1. Calculated effective multiplication factors and relative power values.

| | | $K_{eff}$ | $K_{eff}$ Error (pcm) | Max. ERP (%) | Ave. ERP(%) |
|---|---|---|---|---|---|
| BIBLIS-2D | Momentum weighting functions | 1.02534 | 21 | 1.18 | 0.42 |
| | Galerkin weighting functions | 1.02540 | 27 | 1.45 | 0.62 |
| IAEA-2D | Momentum weighting functions | 1.00420 | 129 | 11.88 | 4.96 |
| | Galerkin weighting functions | 1.00534 | 16 | 8.88 | 3.52 |

## *Conclusions*

Regarding the results, it was concluded that in order to increase accuracy with the acceptable time of computing (4 Seconds for rectangular geometry and 28 seconds for hexagonal geometry with Intel® Core™ i7-4510U Processor), the Momentum weighting function for rectangular geometry and the Galerkin weighting function for hexagonal geometry can be used to discretize equations without reducing the node size.

Therefore, in order to increase the accuracy while maintaining the speed of calculations, without reducing the size of nodes, the appropriate weight function can be used in discretization, which can be very useful in performing calculations of different transients.

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