



Extension of $SU(1,1)$ based transitional Hamiltonian in description of intruder levels of ^{166}Dy

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Abstract

In this paper, the $SU(1,1)$ -based transitional Hamiltonians are extended by adding a 2p-2h excitation term to describe some intruder levels of ^{166}Dy nucleus. We used the quantum numbers of $U(5)$ and $O(6)$ dynamical limits to label different levels of this nucleus and control the effect of each symmetry by a control parameter. Our results for 0_2^+ , 2_2^+ and 0_3^+ intruder levels are in satisfactory agreement with experimental counterparts. Also, the results of this extended formalism improve the theoretical predictions for normal states in comparison with non-perturbed Hamiltonian.

Keywords: intruder states, transitional Hamiltonian, particle- hole excitation.

Introduction

The observation of excited state between the levels of ground band, refer to mixing of symmetries. These levels are intruder states and must describe by such formalism which perturbed by some excitation. The IBM Hamiltonian was written from the beginning in second quantization form in terms of the generators of the unitary Lie algebra $U(6)$. The model assumes that low-lying collective excitations of the nucleus can be described in terms of the number N of s and d bosons. The bosons correspond to pairs of nucleons in valence shell, coupled to angular momentum ($j=0$) s boson ($j=2$) d boson; N is constant for a given nucleus and equal to half its number of valence nucleons. Lie algebra $U(6)$ subtended by s and d bosons. The model presents three special limits that can be solved easily these three limits are $U(5)$, $SU(3)$ and $O(6)$ dynamical symmetry. The study on Dy nucleon reports the excitation of 0_2^+ ,... levels intruder states these levels are classified as two phonon states where must describe by 2p-2h excitation. [3-7]. A method to describe the intruder excitations in the IBM proposed to associate the different shell-model spaces of $0p-0h$, $2p-2h$,... excitation that two or four protons in the $z=50-82$ major shell can be excited to next major shell with the corresponding boson spaces comprising $N, N+2, \dots$ bosons, with $N = N_\pi + N_\nu$ the model Hamiltonian has the form $\hat{H} = \hat{H}_{reg} + \hat{H}_{2p-2h}$ where mixes the regular ($0p-0h$) and ($2p-2h$) configuration. The correlation between valence protons and neutrons is enhanced due to this cross-shell excitation of protons resulting in the lowering of excited 0_2^+ ,... configurations. In this paper we used a $U(5)$ - $SO(6)$ transitional Hamiltonians which are defined in an affine $SU(1,1)$ algebra in IBM1 formalism [1,2]

Theoretical description

The nature of 0_2^+ , 2_2^+ in ^{166}Dy is reported in ref [6] which corresponds with $SO(6)$ symmetry. This nucleus ^{166}Dy which is located near the close proton shell has transitional behavior and we extended $SU(1,1)$ Hamiltonian to describe such levels

with more accuracy to this aim we extended Hamiltonian. It is a commonly used method which adds a corresponding operator and this adds the increase effect of $O(6)$ symmetry on the other hand our $SU(1,1)$ Hamiltonian has a mixing of $U(5)$ and $O(6)$ symmetries.

By employing the generators of $SU(1,1)$ Algebra, the following Hamiltonian is constructed for the transitional region between $U(5) \leftrightarrow SO(6)$ limits [1]

$$H_{extended} = H_{su(1,1)} + H_{so(6)}, \quad H_{su(1,1)} = g S_0^+ S_0^- + \varepsilon S_1^0 + \gamma \hat{C}_2(SO(5)) + \delta \hat{C}_2(SO(3)) \quad (1)$$

$g, \varepsilon, \gamma, \delta$ are real parameters where $\hat{C}_2(SO(3))$ and $\hat{C}_2(SO(5))$ denote the Casimir operators of these groups. It can be seen that Hamiltonian would be equivalent with $SO(6)$ Hamiltonian if $C_s = C_d$ and with $U(5)$ Hamiltonian when $c_s = 0$ & $c_d \neq 0$. Therefore, the $c_s \neq c_d \neq 0$ requirement just corresponds to the $U(5) \leftrightarrow SO(6)$ transitional region. In our calculation we take $C_d(=1)$ constant value and C_s vary between 0 and C_d

Eigenstates of Extended Hamiltonian (1) can be obtained with using the Fourier-Laurent expansion of eigenstates and $SU(1,1)$ generators in terms of unknown c - number parameters x_i with $i = 1, 2, \dots, k$. It means, one can consider the eigenstates as

$$|k; \nu, \nu_n, \Delta, LM\rangle = \sum_{n_i \in Z} a_{n_1} a_{n_2} \dots a_{n_k} x_1^{n_1} x_2^{n_2} \dots x_k^{n_k} S_{n_1}^+ S_{n_2}^+ \dots S_{n_k}^+ |w\rangle \quad (2)$$

Eigenvalues of Hamiltonian (1), i.e. $E^{(k)}$, can be expressed [1]

$$\hat{H}_{so(6)} \equiv \hat{C}_2[SO(6)] = h^{(k)} \quad (3)$$

$$E_{extended}^{(k)} = E^{(k)} + h^{(k)}, \quad E^{(k)} = h^{(k)} + \gamma \nu(\nu + 3) + \delta L(L + 1) + \alpha \Lambda_1^0$$

$$\Lambda_1^0 = \frac{1}{2} [c_s^2 (\nu_s + \frac{1}{2}) + c_d^2 (\nu + \frac{5}{2})], \quad h^{(k)} = \sum_{i=1}^k \frac{\alpha}{x_i} \quad (4)$$

We have solved with definite values of c and α for $i = 1$ to determine the roots of Beth-Ansatz equations (BAE) with specified values of ν_s and ν . We carry out this procedure with different values of c and α to provide energy spectra



(after inserting γ and δ) with minimum variation as compared to the experimental counterparts;

$$\sigma = \left(\frac{1}{N_{tot}} \sum_i |E_{exp}(i) - E_{cal}(i)|^2 \right)^{1/2} \quad (5)$$

which N_{tot} is the number of energy levels ($0_1^+, 2_1^+, 4_1^+, 0_2^+, 2_2^+, 4_2^+$ and *etc.*, e.g. 12 levels up to 2_4^+)

Results

Table 1. The parameters of IBM-1 Hamiltonian $su(1,1)$ for ^{166}Dy All quantities (except c_s) are in keV. σ regards as the quality for extraction processes

| c_s | δ | γ | α | $\bar{\sigma}$ |
|-------|----------|----------|----------|----------------|
| 0.2 | -18.80 | 16.42 | 500 | 320 |
| 0.3 | -18.82 | 15.97 | 500 | 335 |
| 0.4 | -18.85 | 15.34 | 500 | 343 |
| 0.5 | -18.89 | 14.53 | 500 | 351 |
| 0.6 | -18.94 | 13.54 | 500 | 302 |
| 0.7 | -19.00 | 12.36 | 500 | 312 |
| 0.8 | -19.07 | 11.01 | 500 | 328 |
| 0.9 | -19.15 | 9.74 | 500 | 360 |
| 1 | -19.23 | 7.76 | 500 | 365 |

Table 2. The parameters of IBM-1 Extended Hamiltonian $su(1,1)$ for ^{166}Dy isotope All quantities (except c_s) are in keV. σ regards as the quality for extraction processes

| c_s | δ | γ | α | $\bar{\sigma}$ |
|-------|----------|----------|----------|----------------|
| 0.2 | -8.09 | 8.70 | 500 | 228 |
| 0.3 | -8.91 | 8.14 | 500 | 232 |
| 0.4 | -8.92 | 8.17 | 500 | 236 |
| 0.5 | -8.94 | 7.76 | 500 | 241 |
| 0.6 | -8.94 | 7.27 | 500 | 249 |
| 0.7 | -9.00 | 6.68 | 500 | 202 |
| 0.8 | -9.03 | 6.00 | 500 | 225 |
| 0.9 | -9.07 | 5.24 | 500 | 229 |
| 1 | -9.11 | 4.38 | 500 | 238 |



