



## Investigation of Structure and Nuclear Shape Phase Transition in Odd Nuclei in a multi-j model

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### Abstract

In this paper, we study the nature of the dynamics in second-order Quantum Phase Transition (QPT) between vibrational ( $U^{BF}$  (5)) and  $\gamma$ -unstable ( $O^{BF}$  (6)) nuclear shapes. Using a transitional Hamiltonian according to an affine SU(1,1) algebra in combination with a coherent state formalism, Shape Phase Transitions (SPT) in odd-nuclei in the framework of Interacting Boson Fermion Model (IBFM) are investigated. Classical analysis reveals a change in the system along transition in critical point. The role of a fermion with angular momentum j at the critical point on quantum phase transitions in bosonic systems is investigated via semi-classical approach. The effect of the coupling of the odd particle to an even-even boson core is discussed along the shape transition and, in particular, at the critical point.

**Keywords:** Quantum Phase Transition, Interacting Boson Fermion Model, SU(1,1) algebra, Classical analysis.

### Introduction

When the number of nucleons modifies, the quantum shape phase transition happens as that the structure of the system changes from one character to another. The existence of shape phase transitional behavior manifests in the both of even-even and odd-A nuclei [1, 2]. In this paper, a systematic study of spherical to  $\gamma$ -unstable shape phase transition and the effects of coupling a single particle on SPTs in the framework of IBFM is present. Particularly, how the existence of an odd particle can influence SPT is investigated.

### Theoretical Framework: The Applied Transitional Hamiltonian with respect to the Affine SU(1, 1) Algebra

In this section, we study the coupling of an even core that undergoes a transition from spherical U(5) to  $\gamma$ -unstable O(6) situation to a particle moving in the 1/2, 3/2, and 5/2 orbitals by using the affine SU(1,1) Lie Algebra and the Bethe ansatz technique within an infinite-dimensional Lie algebra in the framework of the interacting boson-fermion model [3,4]. To this aim we have used the same formalism to extend IBFM calculation to the case that a j (1/2, 3/2, and 5/2) fermion coupled to a boson core. By employing the generators of Algebra SU(1,1) and Casimir operators of subalgebras, the following Hamiltonian for transitional region between  $U^{BF}$  (5) -  $O^{BF}$  (6) limits is prepared

$$H = gS_0^+S_0^- + \alpha S_1^0 + \delta C_2(O^{BF}(5)) + \gamma C_2(O^{BF}(3)) + \eta C_2(\text{spin}^{BF}(3)) \quad (1)$$

For evaluating the eigenvalues of Hamiltonians Eq.(1) the eigenstates is considered as

$$|k; \nu_s \nu_d n_\Delta JM\rangle = \Theta' S_{x_1}^+ S_{x_2}^+ \dots S_{x_k}^+ |lw\rangle^{BF}, \quad (2)$$

### Energy surface for j=1/2, 3/2, 5/2

Intrinsic states for the mixed boson-fermion system can be constructed by coupling the odd single-particle states to the coherent states of the even core. So, expectation value of  $H_B$  in the boson condensate are given as

$$E = \frac{N_B(N_B-1)}{4(1+\beta^2)^2} \{c_s^2 + 2c_s c_d \beta^2 + c_d^2 \beta^4\} + \frac{\alpha}{4} c_s^2 \left( \frac{2N_B}{1+\beta^2} + 1 \right) + \frac{\alpha}{4} c_d^2 \left( \frac{2\beta^2 N_B}{1+\beta^2} + 5 \right) + 2\delta \frac{\beta^2 N_B}{1+\beta^2} + 12\gamma \frac{\beta^2 N_B}{1+\beta^2} + 12\eta \frac{\beta^2 N_B}{1+\beta^2} \quad (3)$$

By integrating out the boson degrees of freedom, i.e. by taking the expectation value of  $H_F$  and  $V_{BF}$  in the boson condensate, one can obtain the expectation value of the  $H_F + V_{BF}$  in the boson condensate as

$$E = E_B + 4\delta \frac{N_B \beta^2 \cos^2 \gamma}{1+\beta^2} \langle 2020|30\rangle \left\{ \langle K_0^{(3)}(2,2) \rangle + \langle K_2^{(3)}(2,2) \rangle \right\} + \langle K^{(1)}(2,2) \cdot K^{(1)}(2,2) \rangle \\ \times (4\delta + 20\gamma + 20\eta) + 4\delta \langle K^{(3)}(2,2) \cdot K^{(3)}(2,2) \rangle + \eta \left\langle S^{(1)}\left(\frac{1}{2}, \frac{1}{2}\right) \cdot S^{(1)}\left(\frac{1}{2}, \frac{1}{2}\right) \right\rangle \\ - \sqrt{20}\eta \left\langle K^{(1)}(2,2) \cdot S^{(1)}\left(\frac{1}{2}, \frac{1}{2}\right) + S^{(1)}\left(\frac{1}{2}, \frac{1}{2}\right) \cdot K^{(1)}(2,2) \right\rangle \quad (4)$$

$$K_{\mu}^{\lambda}(k, k') = -\sum_s \sum_{j, j'} \sqrt{(2j+1)(2j'+1)} (-1)^{k+s+j'+\lambda} \begin{Bmatrix} j & j' & \lambda \\ k' & k & s \end{Bmatrix} A_{\mu}^{(\lambda)}(j, j')$$

$$S_{\mu}^{\lambda}(s, s') = -\sum_k \sum_{j, j'} \sqrt{(2j+1)(2j'+1)} (-1)^{k+s+j'+\lambda} \begin{Bmatrix} s & s' & \lambda \\ j' & j & k \end{Bmatrix} A_{\mu}^{(\lambda)}(j, j')$$

$$A_{\mu}^{(\lambda)}(j, j') = [a_j^{\dagger} \times \tilde{a}_{j'}]_{\mu}^{(\lambda)} \quad (5)$$

### Results and discussion

The evolution of the energy surfaces along the shape phase transition for the boson core and the odd-even systems with considering different angular momenta 1/2, 3/2, 5/2, orbit are displayed in Figures 2 and 3.

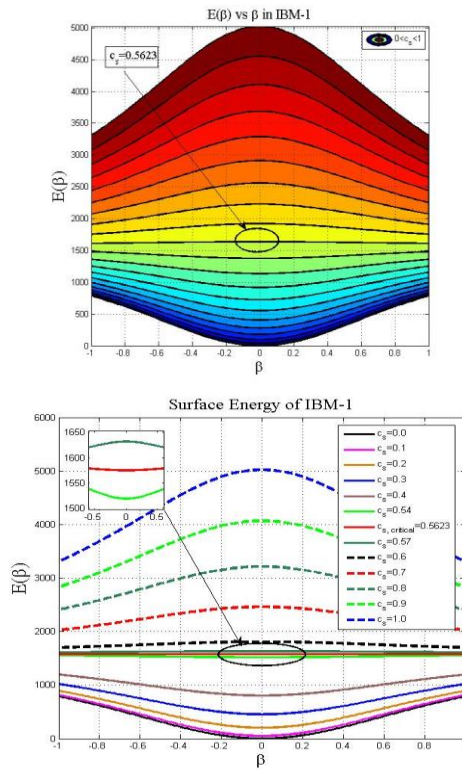


Fig.2 The energy surfaces for an even-even case, as a function of the deformation parameter  $\beta$  for different values of the control parameter  $C_s$ .

In analyzing figures, it can be said that, the evolution of the energy surfaces suggest the SPT critical points  $C_s = 0.56$  and  $0.68$  for even-even and odd-A, respectively. We see from the figures that in these cases the odd particle drives the system toward  $\gamma$ -unstable shape. This gives rise to an effective shift of the critical point. For example, for the odd-A systems, the critical point move to  $C_{s,critical} = 0.68$  (for  $j=1/2, 3/2, 5/2$  orbits), where in fact the corresponding energy surface becomes very flat.

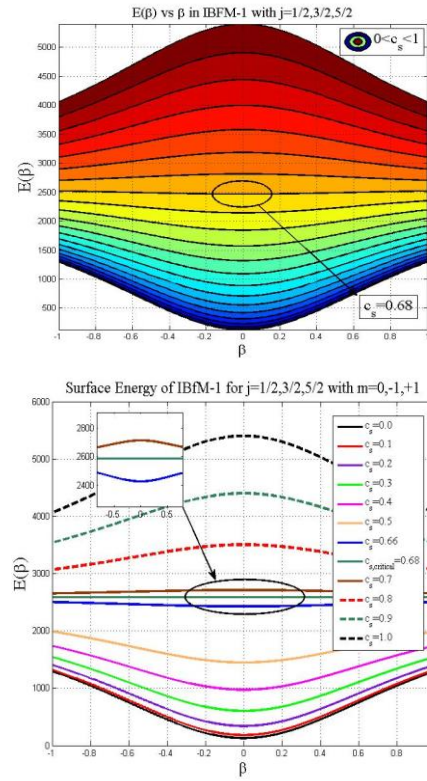


Fig.3. The energy surfaces for an Odd-A nuclei with  $j=1/2, 3/2, 5/2$  as a function of the deformation parameter  $\beta$  for different values of the control parameter  $C_s$ .

### Conclusions

In this paper, the effect of the coupling of a single fermion to a boson core that performs a transition from spherical to  $\gamma$ -unstable shapes, in particular, at the critical point and its around is investigated. The energy surfaces along the shape phase transition is calculated for the boson core and the odd-A systems with considering angular momenta 1/2, 3/2, 5/2. We found the coupling of the single fermion with angular momentum  $j$  to the even-even system doesn't change the geometry imposed by the boson core performing the transition and only the position of the critical point has been shifted by the addition of the odd particle with respect to the even case. So, well known properties in the even-even nuclei persist also in the odd-A system.

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