Statistical fluctuations of E2 transition rates in $^{156-158}$Gd and $^{166-168}$Er isotopes

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Abstract
In this study, we investigated the statistical fluctuations of the electric quadrupole transition probabilities of $^{156-158}$Gd and $^{166-168}$Er. The Porter-Thomas distribution is applied to examine the statistical situation of considered transition rates in comparison with both regular and correlated limits quantitatively. Sequences are prepared by using all the available experimental data of this inter-band of quadrupole transitions between different levels of the ground band. We used the least square method in MATLAB software to extract the parameter of the Porter-Thomas distribution function in these sequences. The results suggest correlated statistics for both $^{166-168}$Er isotopes which are located in the middle of closed proton shells and classified as the prolate deformed nuclei which have axial symmetry. Also, for the Gd isotopes, our results show a deviation from the GOE limit and a regular behavior in their transitions. This uncorrelated behavior is significant for the ^{156}Gd isotope and can relate to the gamma-unstable nature of this isotope.
Keywords: Porter-Thomas distribution; quadrupole transition rates; statistical analysis; experimental data.

Introduction
Random Matrix Theory (RMT) is used for the statistical study of nuclear systems [1,2]. Due to the quantum nature of the nuclear systems and the uncertainty in measuring the nuclear observables, such statistical approaches to the nuclear systems is of particular importance [3-5].
The first statistical fluctuations of electric quadrupole transitions probability by using RMT is related to a study by A. Adams et al. [6] on electromagnetic bipolar and quadrupole transition probabilities in the Na^{22} nucleus. The results showed the transition probabilities' statistical behavior's adherence of Porter-Thomas Distribution [7], which conforms to the Gaussian distribution in RMT.
In this study, we used a parametric approach that employs the Porter-Thomas distribution and classifies the statistical behavior of different systems with its quantity. We aim to evaluate the statistical fluctuations of electric quadrupole transition probabilities for selected isotopes. To do this, we used the available experimental data to prepare sequences.

Theoretical description
The Porter-Thomas distribution function is as follows:

$$P_\nu(y) = \left(\frac{\nu}{2\langle y\rangle}\right)^{\frac{\nu}{2}} \frac{y^{\frac{\nu}{2}-1} exp\left(\frac{-\nu y}{2\langle y\rangle}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \quad , \qquad (1)$$

where y represents the unfolded values of the quadrupole transition rates. There are ways to unfold and get y in [8]. Due to the wide range of data changes, y intensities should be normalized by dividing them by their smooth part $\bar{y}(E,E')$:

$$y_{fi} \equiv B(E2; i \to f) = \frac{|\langle f|\hat{T}(E2)|i\rangle|^2}{\bar{y}(E = E_i, E' = E_f)} \quad , \qquad (2)$$

Then, using Porter-Thomas statistical methods estimation method, we study the y obtained above.
Parameter ν is obtained by fitting the distribution function to the data histogram. The closer the value of the variable ν is to zero, the more regular the system is [8]. For better evaluation, the $y \to \log y$ conversion is usually used first for the data, and then to get the value of ν, the histogram of the normalized data is fitted by the interpolation function given in Equation 1 after converting $P_\nu(y) \to P_\nu(\log y)$ to obtain the value of ν [8,9].
It is the value of quantity ν that determines the Porter-Thomas distribution of data close to the Poisson $(P(s) = e^{-s})$ or Gaussian $\left(P(s) = \frac{\pi s}{2} \exp\left(\frac{\pi s^2}{4}\right)\right)$ distribution [10].

Results and discussion
In this paper, the intended data of non-similar symmetry and from all bands and energy levels of different isotopes are selected. The ^{156}Gd, ^{158}Gd, ^{166}Er, and ^{168}Er isotopes are good options for examining the electric quadrupole transition probabilities because there are detailed experimental data about them [11-14].
For statistical investigation of these isotopes, all quadrupole transition probabilities of them are selected and their statistical distribution is plotted correspondingly. The number of data for ^{156}Gd, ^{158}Gd, ^{166}Er, and ^{168}Er isotopes is 42, 30, 27 and 31, respectively.

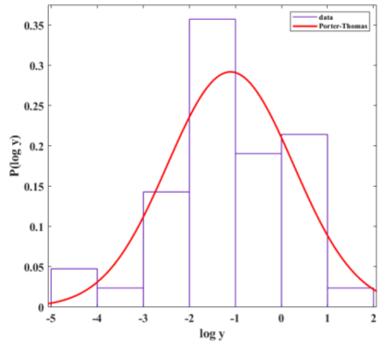

Fig.1. Statistical distributions for B(E2) transition probabilities in the ^{156}Gd nucleus.

The Porter-Thomas distribution result indicates that by ν=0.4218, the system is semi-regular.

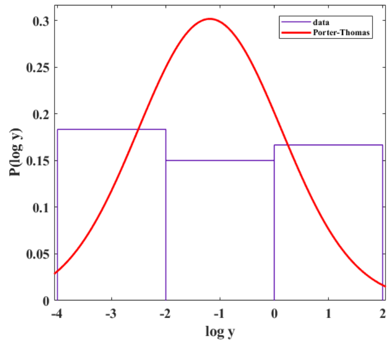

Fig.2. Statistical distributions for B(E2) transition probabilities in the ^{158}Gd nucleus.

The Porter-Thomas distribution result indicates that by ν=0.6324, the system is semi-chaotic.

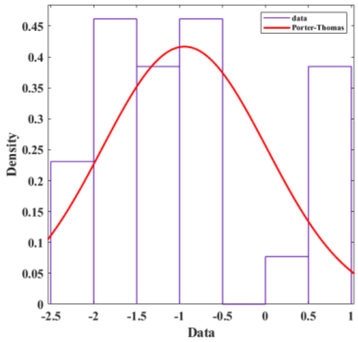

Fig.3. Statistical distributions for B(E2) transition probabilities in the ^{166}Er nucleus.

The Porter-Thomas distribution result indicates that by ν=0.9575, the system is chaotic.

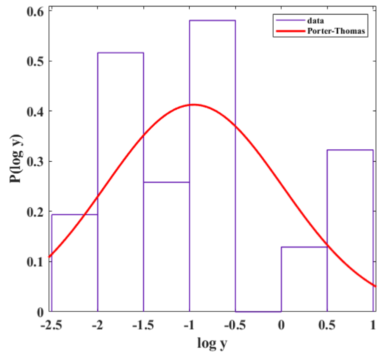

Fig.4. Statistical distributions for B(E2) transition probabilities in the ^{168}Er nucleus.

The Porter-Thomas distribution result indicates that by ν= 0.9572, the system is chaotic.

Conclusions

In this study, the intended data were collected from all bands and energy levels of ^{156}Gd, ^{158}Gd, ^{166}Er, and ^{168}Er isotopes. In this paper, the Porter-Thomas methods were used for the statistical analysis of electric quadrupole transition probabilities in selected isotopes. Different sequences were prepared with available experimental data. To describe the statistical situation of the chosen sequences compared to two chaotic and regular limits, first, the Porter-Thomas distribution of data was fitted to the histogram of each sequence using the least square fitting method, then, The value of ν obtained was used to determine the correlated or independent behavior of the data.

The Porter-Thomas distribution result indicates that by ν=0.4218, 0.6324, 0.9575, and 0.9572; the ^{156}Gd is semi-regular, the ^{158}Gd is semi-chaotic, the ^{166}Er is chaotic, and the ^{168}Er is chaotic, respectively.

The results suggest correlated statistics for both $^{166\text{-}168}$Er isotopes which are located in the middle of closed proton shells and classified as the prolate deformed nuclei which have axial symmetry. Also, for the Gd isotopes, our results show a deviation from the GOE limit and a regular behavior in their transitions. This uncorrelated behavior is significant for the ^{156}Gd nucleus and can relate to the gamma-unstable nature of this nucleus.

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