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Study of interaction between Skyrmions using Rebbi and Jacobs variational method

Ataei M. [1], Lookzadeh H. [1], Tatari M. [1*]

[1] Department of Physics, Faculty of Science, Yazd University, P.O. Box: 8915818411, Yazd, Iran

[*] *Email: mtatari@yazd.ac.ir*

Abstract
Skyrmions are a type of soliton which first has been introduced by Skyrme, for the explanation of nuclear interaction. We will explain about Skyrmions and try to numerically solve its equation using Rebbi and Jacobs variational method. We compare the energy of the interaction of two skyrmions that are far apart each other and when they merge into each other. A repulsive interaction is obtained which is in accordance of dipole approximate method.
Keywords: Nuclear model, Skyrmion, Skyrme, Topological Soliton, variational method

Introduction
Solitons are a type of especial wave-like structure in physics. The dispersive property of these waves is neutralized with nonlinear behavior, so solitons preserve their permanent form. They are localized within a region. These structures have many attractive properties in different area of physics. For example, magnetic penetration type in superconductor of type 2 obeys vortex solution, which is a type of soliton with cylindrical symmetry. Solitonic optical pulses are used in fiber optic to prevent the attenuation. One of the soliton's interesting models is the one that is proposed by the England nuclear physicist Skyrme to describe the behavior of nuclear and baryon matter. The Skyrme model has been existed before the existence of the strong nuclear force interaction and the quark model. This model does not have all the features of soliton structures, but Skyrmion has soliton properties. One of the most important soliton structures used for baryons is called Skyrmion [1]. Nuclear interaction can be explained better considering Skyrmion [5].

Skyrmion
Skyrmions are the solution to the nonlinear equations that can describe the behavior of internal nuclear of material, and because of the nonlinear behavior, mankind hasn't fully identified the dynamics of Skyrmion. This model is based on the nonlinear quantum chromo dynamics (QCD) model for the Pion meson. Finally, the Skyrmion equation becomes a second-order nonlinear differential equation, and because it is a nonlinear equation, it cannot be solved analytically [2]. Following details in [4] the Skyrmion equation is as follows:

$$(r^2 + 2\sin^2 f)f'' + 2rf' + \sin 2f\,(f'^2 - 1 - \frac{\sin^2 f}{r^2}) = 0.$$

f is the Skrmion function and r is distace from the origin. Numerical solution, like the shooting parameter method, can be used to solve it. In addition, it can be used the Rebbi and Jacobs variational method to find the equation numerically. Topological charge conservation can be used instead of charge conservation to study the interaction between these structures. Charge conservation comes from Nother theorem while topological charge conservation is due to the geometry. Emergence of a new charge conservation leads to the availability of studying the interaction between these soliton structures.

Rebbi and Jacobs method
The Variational method proposed by Rebbi and Jacobs [3] is one of the common methods for obtaining solutions of soliton structures. We want to find the solution of Skyrmion equation with this method. By knowing the solution behavior of r = 0 and r→∞, an acceptable function can be suggested, so that in addition to knowing the behavior of the function within these limits, we can; Find the behavior of the function in the middle region for each distance range. According to the limit of the solution, we multiply a polynomial by the exponential function and accept it as a solution:

$$f(r) = \sum_{i=0}^{-n} C_i r^i \exp(r)\,,$$

We find the polynomial coefficients from the asymptotic form of the function and the rest of the remaining coefficients from the variational method. Up to power six are used for the numerical solution. In this method, to solve the equations with the help of their asymptotic behavior, we propose virtual functions that minimize these free energy functions, and we accept these functions as solution of the equation [4]. Figure (1) shows the obtained solution of the Skyrmion equation by the variational method and as a numerical solution. Our numerical solution obeys the asymptotic values at zero and large distances. Also, with this method, in Figure (2), we have drawn a Lagrangian density diagram for a Skyrmion, and the area below this diagram gives us the amount of energy of a Skyrmion, and this value is equal to 7.086.

In this figure r has the dimension of length and L has the dimension of energy density. Our numerical solution gives a finite energy for the Skyrmion which is the main property of a soliton structure. Figure (3) shows the Lagrangian density for the winding n=2. The number of maximum in the free energy density for n=2 should be two and again this is clear in the obtained figure from numerical solution. It's mass or energy is equal to 22.356. Energy of two Skyrmion with n=1 is equal to 14.172. So the energy of two separate Skyrmion at large distance is lower than a Skyrmion with n=1 at zero distance. Using Abrikosov ansatz one can obtain the behavior of interaction between solitonic structure when they are far from each other. Due to this method if the energy of two separate Skyrmion with winding number n=1 is larger (smaller) than the energy of a Skyrmion with n=2, then the interaction is atraction (repulsion). According to the Abriksove ansatz our results shows a repulsive interaction between Skyrmion.

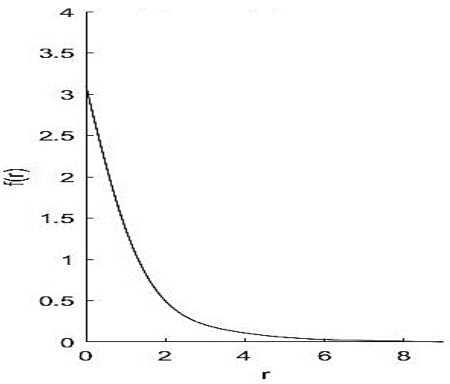

Figure (1): f(r) profile for a Skyrmion with n=1 using the variational method.

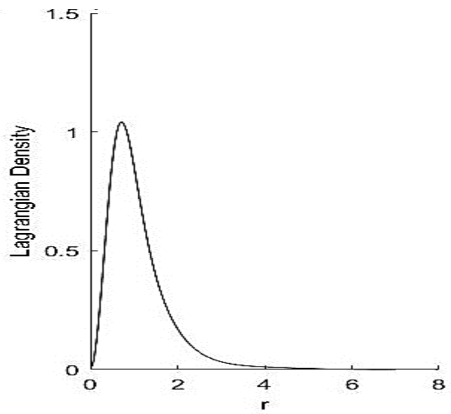

Figure (2): Energy density for a Skyrmion with n=1 using the variational method.

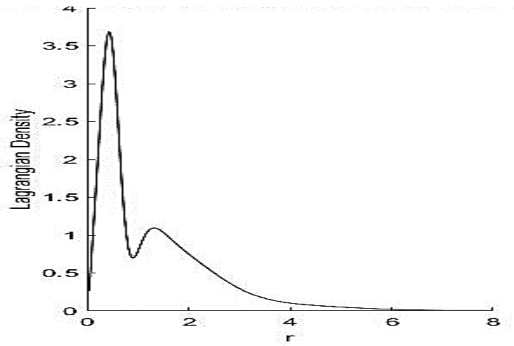

Figure (3): Energy density for a Skyrmion with n=2 using the variational method.

Conclusions

Here we introduce the Skyrmions and explain the forms of solving these equations. Explaining the Rebbi and Jacobs method, we used this method to solve the Skyrmion equation numerically and compare the interaction energy of two Skyrmions at the large distance. This leads a repulsive interaction between skyrmions at large distance.

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