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Studying ${}_{\Lambda\Lambda}^{6}He$ hypernuclei by modern ΛΛ potential derived from Lattice QCD Simulations

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Abstract

In this work, we examine the first principle lattice quantum chromodynamics (LQCD) $\Lambda\Lambda$ interactions derived by HAL collaboration, which are the most consistent potential with the *Large Hadron Collider* (*LHC*) and A Large Ion Collider Experiment (ALICE) data to study the ground state properties of the ${}_{\Lambda\Lambda}^{6}$He hypernucleus. Theoretically, the ${}_{\Lambda\Lambda}^{6}$He system is considered as an α particle and two Λ hyperons. Since the binding energy of α nucleus is relativly large, it can be considered as solid particle, so its excited states can not play a role in the desired energy. In addition we have calculated the matter radious of ${}_{\Lambda\Lambda}^{6}$He hypernuclei. Furthermore, in order to have better comparisons we did calculations by the Nijmegen model D hard-core interaction (ND) for $\Lambda\Lambda$ phenomenological potential which often used in hypernuclei studying. For $\Lambda\alpha$ potential the famous two-range Gaussian (Isel-type) interaction is used.

Keywords: ${}_{\Lambda\Lambda}^{6}$He hypernuclei, HAL QCD ΛΛ potential, three-body interactions

Introduction

In order to fully understand the baryon-baryon interaction, it is important to obtain information about the $\Lambda\Lambda$ interaction in the $S = -2$ sector [1,2,3]. The observed Λ-hypenuclei provide information about the structure of the nucleus when a Λ particle is added to a normal nucleus. In addition to advances in data and experimental techniques, recently the ΛΛ interacting potentials at nearly physical quark masses ($m_k \approx 525$ MeV و $m_\pi \approx 146$ MeV) has been calculated in the lattice QCD simulations by the HAL QCD Collaboration. The potentials are obtained on a large space-time volume $L^4 = (8.1\, fm)^4$ with a lattice spacing $a = 0.0846\, fm$ [4]. We have studied the ground-state properties of double hyperon ${}_{\Omega\Omega}^{6}$He in a three-body model ($\Lambda + \Lambda + \alpha$) by solving the coupled Faddeev equations with the hyperspherical harmonics expansion method [5,6]. HAL QCD $\Lambda\Lambda$ interactions are the most consistent potential with the LHC ALICE data. Our results are consistent with the existing theoretical and experimental data[7].

ΛΛ potential (phenomenological ND model)

The ND $\Lambda\Lambda$ potential [8] assuming the same hard core for the NN and $\Lambda\Lambda$ potentials in the ${}^{1}S_{0}$ channel. This two-body potential in configuration space in ${}^{1}S_{0}$ channel are used as input to the Faddeev equations [6] which has three-range Gaussian form and it is depicted in Fig.1 [9,10].

$$V_{\Lambda\Lambda}(r,\gamma) = \sum_{i=1}^{3} \upsilon^{(i)}(r,\gamma) \exp\left(-\frac{r^2}{\beta_i^2}\right). \qquad (1)$$

Table 1. Parameters in Eq.(1).

I	$\upsilon^{(i)}(MeV)$	$\beta_i(fm)$
1	−21/49	1/342
2	−379/1	0/777
3	9324	0/350

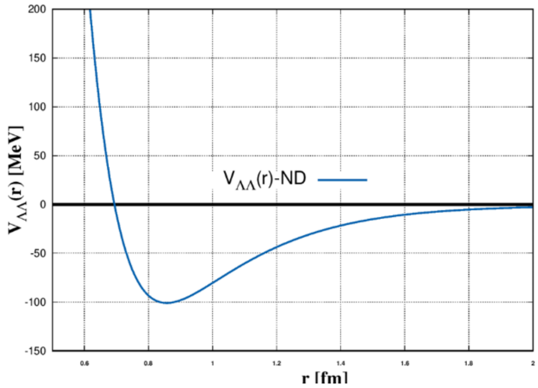

Figure **1.** The ND type ΛΛ potential in ${}^{1}S_{0}$ channel.

ΛΛ potential (HAL QCD)

For $\Lambda\Lambda$ interaction extracted from lattice QCD, we have used the two-body lattice resultant potentials developed by HAL QCD Collaboration in ${}^{1}S_{0}$ channel [4],

$$V_{\Lambda\Lambda}(r) = \sum_{i=1}^{2} \alpha_i \exp(-r^2/\beta_i^2) + \lambda_2 (1 - \exp(-r^2/\rho_2^2))^2 (\frac{\exp(-m_\pi r)}{r})^2. \qquad (2)$$

The parametes in Eq. (2) for three $\Lambda\Lambda$ potential of HAL model (at lattice intervals of $t/a = 11$ (i), $t/a = 12$ (ii) and $t/a = 13$ (iii)) are given in Table 2. In Fig.2, we compare the HAL QCD ΛΛ potential at different lattice time slices. By having two-body $\Lambda\Lambda$ and $\Lambda\alpha$ potentials as input we solved three-body problem $\Lambda + \Lambda + \alpha$ by solving Faddeev coupled equations in hyperspherical harmonics method (HH) :

Table 2. presents potential parameter $V_{\Lambda\Lambda}(r)$ of Eq.(2).

t/a	$\alpha_1(MeV)$	$\beta_1(fm)$	$\alpha_2(MeV)$	$\beta_2(fm)$	$\lambda_2(MeV.fm^2)$	$\rho_2(fm)$
11	1466.4	0.160	407/1	0/366	−170/3	0/918
12	1486.7	0.156	418/2	0/367	−160/0	0/929
13	1338.0	0/143	560/7	0/322	−176/2	1/033

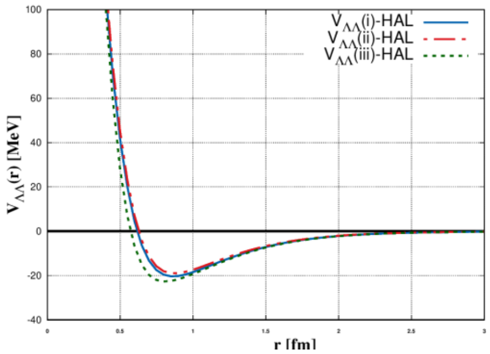

Figure 2. Three HAL models of ΛΛ potential in the channel $^1S_0$, Eq.(2) by the parameters given in Table 2.

Table 3. Dependency of the ground state binding energy B_3 and radious of matter r_{mat} of $_{\Lambda\Lambda}{}^6He$ are compared for two type potential, i.e., HAL and ND ΛΛ potential in HH method.

K_{max} (i_{max} = 35)	B_3(MeV) HAL	r_{mat}(fm) HAL	B_3(MeV) ND	r_{mat}(fm) ND
5	6/897	1/953	−5/525	2/060
10	7/321	1/948	−8/190	1/873
15	7/404	1/951	−8/846	1/852
20	7/446	1/953	−9/169	1/847
25	7/456	1/954	−9/241	1/847
30	7/463	1/955	−9/289	1/847
35	7/465	1/955	−9/305	1/847
40	7/466	1/955	−9/318	1/847
45	7/467	1/955	−9/323	1/847
50	7/467	1/955	−9/326	1/847
55	7/467	1/955	−9/328	1/848
60	7/468	1/955	−9/329	1/848
65	7/468	1/955	−9/329	1/848
70	7/468	1/955	−9/329	1/848
75	7/468	1/955	−9/329	1/848
80	7/468	1/955	−9/329	1/848

$$(T-E)\Psi_i^{JM} + V_{jk}\left(\Psi_i^{JM} + \Psi_j^{JM} + \Psi_k^{JM}\right) = 0, \qquad (3)$$

where E is the total energy, T is the kinetic energy operator and $\{i,j,k\}$ is a cyclic permutation of {1,2,3}. The total wavefunction Ψ^{JM} of the three-body system is given as a sum of three components Ψ_i^{JM}, $\Psi^{JM} = \sum_{i=1}^{3}\Psi_i^{JM}(x_i,y_i)$, each expressed in terms of one of the three different sets of Jacobi coordinate (x_3,y_3) illustrated in Fig. 3. The HH method is fully described in Refs.[6,5].

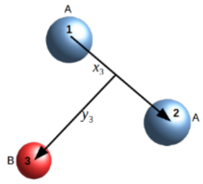

Figure 3.*The Jacobi T-coordinate system used here. The letters A and B denot Λ and α particles, respecively.*

Results and discussion

The ND model, which is a phenomenological potential, is much deeper than the HAL ΛΛ potential model, while the HAL potential is more real. The ground state binding energy B_3 and the radius of the nucleus r_{mat} for ND and HAL ΛΛ potentials in HH method as a function of hypermomentum K_{max} and number of basis used in radial part of wave functions i_{max} are given in Table (3). The parameters K_{max} and i_{max} are defined in Ref.[6] please see this Ref for more detailed description of used method. As can be seen from Table 3 the ground state binding energy for HAL potential is much closer to experimental values $7.25 \pm 0.19\ MeV$ [11].

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