



Investigation of hypothetical $\Omega\Omega\alpha$ hypernuclei in cluster model

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Abstract

Even though lots of Λ -hypernuclei have been found and measured, multi-strangeness hypernuclei consisting of Ω are not yet discovered. The HAL QCD Collaboration in the lattice QCD simulations, have found attractive interaction in the ΩN^5S_2 and $\Omega\Omega^1S_0$ channels which support bound state with a central binding energy of 1.54 MeV and 1.6 MeV, respectively. Motivated by the above, in the present work, we study hypothetical multi-strangeness nucleus ${}_{\Omega\Omega}^6\text{He}$ in the three-body $\Omega\Omega\alpha$ system searching for deeply bound states or resonances which may be sought experimentally. We calculate the binding energy of the multi-strangeness system $\Omega\Omega\alpha$ about 67 MeV. Also, we define the geometrical characteristics of ${}_{\Omega\Omega}^6\text{He}$ ground state i.e. the different r.m.s. distances between the articles including the r.m.s. matter radius.

Keywords: ΩN and $\Omega\Omega$ potentials, hypernuclei, three body cluster interactions

Introduction

The importance of an exact treatment of hypernuclei could help to better understanding of the hyperon-hyperon and hyperon-alpha interactions. Here, we have studied the ground-state properties of double hyperon ${}_{\Omega\Omega}^6\text{He}$ in a three-body model ($\Omega + \Omega + \alpha$) by solving the coupled Faddeev equations with the hyperspherical harmonic expansion method. This abstract takes modern hyperon-hyperon interactions obtained from lattice QCD simulations to calculate the ground state binding energies and geometrical properties of mentioned multi-strangeness hypernuclei. We study the ${}_{\Omega\Omega}^6\text{He}$ system for the first time, and to the best of our knowledge, there is no other study on this system. Our results could provide valuable information for future experimental investigations.

Two-body potentials

The $\Omega\Omega$ potential in 1S_0 channel studied by the HAL QCD Collaboration with a large volume and nearly the physical pion mass [3]. The resultant potential has been fitted by means of an analytic function composed of three Gaussian terms (see Fig. 1)

$$V_{\Omega\Omega}(r) = \sum_{j=1}^3 c_j \exp\left(-\frac{r}{d_j}\right)^2. \quad (1)$$

where the potential parameters, without considering the statistical errors, are $(c_1, c_2, c_3) = (914, 305, -112)$ MeV and $(d_1, d_2, d_3) = (0.143, 0.305, 0.949)$ fm. The $\Omega\alpha$ interaction has also been recently extracted by using a single-folding potential method, based on a separable HAL QCD collaboration Ω -nucleon potential from Lattice QCD in [4] and it is shown in Fig. 2. The $\Omega\alpha$ potential $V_{\Omega\alpha}(r)$ is parametrized in phenomenological Woods-Saxon-type form:

$$V_{\Omega\alpha}(r) = -61 \left[1 + \exp\left(\frac{r - 0.47}{1.7}\right) \right]^{-1}. \quad (2)$$

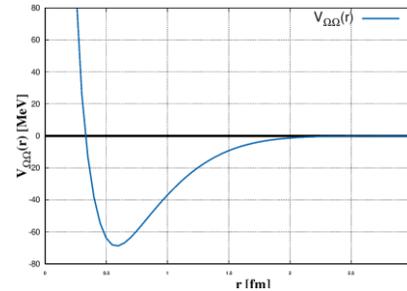


Figure 1. $\Omega\Omega$ potential.

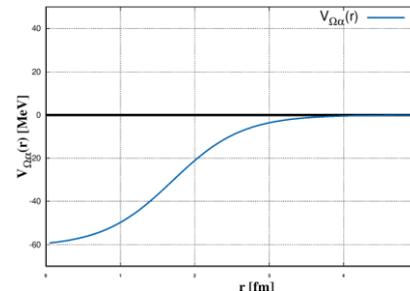


Figure 2. $\Omega\alpha$ potential.

Borromean systems such as ${}^6\text{He}$ have been well modeled as three body (core+N+N) systems and studied in hyperspherical harmonics (HH) method [5, 6]. For a three-body systems where full scale interactions plays role between all three particles; this type of system is the most appropriate for the HH method, owing to the fact that the approximate democratic symmetry (connected with the interchange of positions of particles) which is a feature of the HH method. An outstanding feature of the HH formalism is that the wave function is concentrated in only few parts for

bound states and low continuum energies which correspond to the lowest momenta and energy configurations of the three-body system [7]. For the detailed method which we have used please see Ref. [5].

Results and discussion

Theoretically, $YY\alpha$ system is considered as an α particle and two hyperons as it is shown in Fig.3. Since the α core is firmly bound, it can be considered inert, therefore its excited states could not have any role at the energies of interest. In the case of more complex exotic nuclei, this assumption and simplification may not be reliable. As mentioned implicitly the prominent property of HH calculations is a very fast convergence of the WF, and relatively slow one for the binding energy. In the HH method we can only estimate the binding energy from the asymptotic approach to convergence [5]. The size of model space in which the three-body wave functions are expanded is truncated by fixing a maximum value of the hypermomentum K_{max} . The ground-state energies B_3 of the $\Omega\Omega\alpha$ system is obtained by using the models suggested above in hyperspherical harmonics method and they are given in Table 1 and the geometrical properties are presented in Table 2.

Table 1. The three-body ground state binding energies B_3 (in MeV) calculated for systems $\Omega\Omega\alpha$ within the HH Method.

| system | $B_2(\Omega\Omega)$ | $B_2(\Omega\alpha)$ | B_3, HH | $B_3 (V_{\Omega\Omega} = 0)$ |
|----------------------|---------------------|---------------------|-----------|------------------------------|
| $\Omega\Omega\alpha$ | -1.408 | -22.010 | -67.213 | -48.956 |

Table 2. The expectation values of Jacobi coordinates in $\Omega\Omega\alpha$ systems (Fig.3) in fermi. $R_{\Omega\Omega}$ is the separation between identical hyperons, $R_{\Omega\alpha}$ is the separation between $\Omega\alpha$ pairs, $R_{(\Omega\Omega)\alpha}$ is the separation between the center of mass of $\Omega\Omega$ pair and the spectator α particle. $\langle\rho^2\rangle^{1/2}$ is r.m.s. matter radius of the three-body system containing only point particles, and R_m is the r.m.s. matter radius.

| system | (Spin-Isospin) | $R_{\Omega\Omega}$ | $R_{(\Omega\Omega)\alpha}$ | $R_{\Omega\alpha}$ | $\langle\rho^2\rangle^{1/2}$ | R_m |
|----------------------|----------------|--------------------|----------------------------|--------------------|------------------------------|-------|
| $\Omega\Omega\alpha$ | (0,0) | 1.52 | 1.05 | 1.269 | 2.037 | 1.326 |

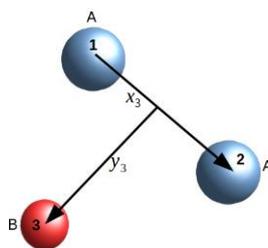


Figure 3- Jacobi coordinate for $\Omega\Omega\alpha(AAB)$ systems.

Conclusions

Recently, the interaction potentials $\Omega\Omega(^1S_0)$ and $N\Omega(^5S_2)$ with quark mass close to its physical value

($m_k \approx 525$ MeV and $m_\pi \approx 146$ MeV) from lattice chromodynamic theory (LQCD) by HAL QCD has been extracted. Cluster formation in many light cores allows the multi-particle problem. In addition to advances in data and experimental techniques, the theoretical efforts of the HAL QCD Collaboration. have reached the point of extraction of the baryon-baryon interaction near the physical bond mass. Their latest results indicate the existence of shallow connection states in ΩN and $\Omega\Omega$ systems.

The results show that the $\Omega\Omega$ interaction is generally attractive, especially the extracted potential, which consists of three Gaussian sentences. In this work, the energy dependence of $\Omega\Omega\alpha$ system is estimated to be about 67 MeV, respectively. Also, the geometrical characteristics of the ground state ${}^6_{\Omega\Omega}\text{He}$ are defined as the distances of r.m.s between the components of the system, in addition to the radius of matter.

References

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