Calculation of magnetic moments of Λ-hypernuclei $^{13}_{\Lambda}C$, $^{17}_{\Lambda}O$ *and* $^{41}_{\Lambda}Ca$

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Abstract

The magnetic moments of Λ-hypernuclei are the most interesting observables which provide a highly sensitive probe of lambda in the hypernuclei structure and also supply direct information on hyperon-nucleon interactions. In this work, we derive the magnetic moments of Λ-hypernuclei such as $^{13}_{\Lambda}C$, $^{17}_{\Lambda}O$ and $^{41}_{\Lambda}Ca$ employing a relativistic approach in the presence of the Dirac equation and the spin-orbit potential in their ground and excited states, i.e. the $1S_{1/2}$, $1P_{3/2}$ and $1P_{1/2}$ states. We, then, extract an analytic solution for the wave function of hyperon which is needed for computing the magnetic moments of Λ-hypernuclei. The hypernuclei magnetic moments are the magnetic moment of the last unpaired baryon for the odd mass hypernuclei, therefore, in our work we study the hypernuclear magnetic moment with one Lambda added to a closed-shell core of nucleons. Since Λ-hypernuclei is an isoscalar particle it is possible to directly probe the modified core current electromagnetically.

Keywords: Lambda hypernuclei, Hyperon, Dirac magnetic moment, Anomalous magnetic moment.

Introduction

Hypernuclei are complicated nuclear systems where one or more nucleons are replaced by one or more hyperons. Hypernuclei can be described by their properties such as the binding energy, the magnetic moment and etc. Among these quantities, the hypernuclei magnetic moments can provide direct information on hyperon-nucleon interaction and the role of hadrons in the nuclear medium [1]. Among the studies performed on the hypernuclei magnetic moments, the following can be mentioned: in Ref [2] new formalism for the magnetic momenta of nucleons and hyperons inside hypernucleus is obtained. In Ref [3] Deviation from the Schmidt value for the magnetic moments of mirror hypernuclei is reviewed.In Ref [4] Magnetic moments of Λ hypernuclei within the time-odd triaxial relativistic mean-field approach are investigated. In this work, in a relativistic approach, we analytically determine the magnetic moment of several Λ-hypernuclei such as $^{13}_{\Lambda}C$, $^{17}_{\Lambda}O$ and $^{41}_{\Lambda}Ca$ in their ground and excited states, i.e. the $1s_{1/2}$, $1p_{3/2}$ and $1p_{1/2}$ states. For our analysis, we considerin the Dirac equation and the spin-orbit potential and determine the wave function of hypernuclei. Then we compute the Dirac, the anomalous and the total magnetic moments of the ground and excited states of Λ-hypernuclei.

Methodology

The Dirac equation for spin-1/2 particles in the natural units (where $\hbar = c = 1$) is given by

$$\left[\vec{\alpha}.\vec{p}+\beta\left(M+U_S(r)\right)-E+U_V(r)\right]\frac{1}{r}\begin{pmatrix} F_{nk}(r) \\ iG_{nk}(r)\end{pmatrix}=0, \qquad (1)$$

where, β and $\vec{\alpha}=(\alpha_1,\alpha_2,\alpha_3)$ are the Dirac matrices. p and E refer to the momentum and relativistic energy of particles. F(r) and G(r) are wave functions. "n" is the radial quantum number and $k(=\pm(j\pm 1/2))$ (j total angular-momentum) is the possible eigenvalue of the operator $\hat{K}(=\beta(\hat{\sigma}.\vec{L}+1))$. Here, U_S and U_V are the scalar and the vector potentials, respectively. M is the baryon mass. Putting β and $\vec{\alpha}$ in Eq. (1), two coupled first-order Dirac equations are obtained

$$F''_{nk}(r)+k\,r^{-1}F_{nk}(r)=(M+E-\Delta)G_{nk}(r), \qquad (2a)$$

and

$$G''_{nk}(r)-kr^{-1}G_{nk}(r)=(M-E+\Sigma)F_{nk}(r), \qquad (2b)$$

where $\Delta(r)$ and $\Sigma(r)$ are

$$\Delta(r)=U_V(r)-U_S(r) \text{ and } \Sigma(r)=U_V(r)+U_S(r). \quad (3)$$

We consider the case of exact spin symmetry for which $\Delta=0$ and $\Sigma(r)=V_{so}(r)$. The spin-orbit potential $V_{so}(r)$ is

$$V_{so}(r)=-U_1(1-2ZA^{-1})e^{\frac{r-R}{a}}(1+e^{\frac{r-R}{a}})^{-2}r^{-1}\times a^{-1}U_{so}\left[j(j+1)-\ell(\ell+1)-s(s+1)\right] \quad (4)$$

where, Z and A are the atomic and the mass numbers, respectively, and $R=r_0A^{1/3}$. The parameter 'a' refers to the thickness of surface. The values used for the parameters are $r_0=1.2\,fm$, $U_1=-0.075\,MeV$, $U_{so}=3.7516\,MeV$ and a=0.54fm. To solve Eq. (1) for determination of the wave function, we apply the NU (Nikiforov–Uvarov) approach introduced in Ref. [5]. In this method is an appropriate approach to solve the second order differential equations. Considering the general form of a second-order differential equation with an arbitrary potential, as

$$\{\frac{d^2}{dr^2}+\frac{t_1-t_2r}{r(1-t_3r)}\frac{d}{dr}+\frac{1}{[r(1-t_3r)]^2}[-\sigma_1s^2+\sigma_2s-\sigma_3]\}\psi(r)=0 \quad (5)$$

according to the NU approach, the eigenfunction is as

$$\psi(r)=r^{(1-t_1)/2+\sqrt{(1-t_1)^2/4+\sigma_3}}(1-t_3r)^{-(1-t_1)/2+\sqrt{(1-t_1)^2/4+\sigma_3}-\frac{(t_2-2t_3)}{t_3}}$$
$$\times P_n^{\left(t_1+(1-t_1)+2\sqrt{(1-t_1)^2/4+\sigma_3}-1,\frac{(t_2-2t_3)}{t_3}-t_1+(1-t_1)+2\sqrt{(1-t_1)^2/4+\sigma_3}-1\right)}(1-2t_3r) \quad (6)$$

where P is the Jacobi polynomials. Applying the Pekeris approximation in Ref [6] on Eq. (2a) and then using the NU method, we obtain wave function as

$$F_{nk}(r)=N_0(-e^{r/a})^{N_3}(1+e^{r/a})^{-N_3-\frac{N_4}{N_3}}P_n^{\left(N_1-1,\frac{N_2}{N_3}-N_1-1\right)}(1+2e^{r/a}), \quad (7)$$

wehre N_o is the normalization constant and

$$N_1=1+2\sqrt{\beta_3},\ N_2=2e^{\frac{R}{a}}(1+\sqrt{\beta_3})+2\sqrt{-\beta_2e^{\frac{R}{a}}+\beta_3e^{\frac{2R}{a}}+\frac{1}{4}e^{\frac{2R}{a}}+\beta_1} \quad (8a)$$

$$N_3=\sqrt{\beta_3},\quad N_4=-\frac{1}{2}e^{\frac{R}{a}}-\sqrt{-\beta_2e^{\frac{R}{a}}+\beta_3e^{\frac{2R}{a}}+\frac{1}{4}e^{\frac{2R}{a}}+\beta_1}-e^{\frac{R}{a}}\sqrt{\beta_3} \quad (8b)$$

and using Eq. (2a) and (5),G(r) is obtained as

$$G_{nk}(r)=N_0\frac{(-e^{-\frac{r}{a}})^{N_3}(1+e^{-\frac{r}{a}})^{-N_3-N_4}}{a(2M-E)}\Big\{-((n-1)+N_2)e^{r/a}P_{n-1}^{(N_1,N_2-N_1)}(1+2e^{-\frac{r}{a}})$$
$$-\left(\frac{\ell a}{\omega_0}e^{-\frac{r}{a}}+(N_3+N_4)(1+e^{-\frac{r}{a}})^{-1}+N_3\right)P_n^{(N_1-1,N_2-N_1-1)}(1+2e^{-\frac{r}{a}})\Big\}. \quad (9)$$

We use the obtained F(r) and G(r) to calculate the magnetic moment. The magnetic moment is defined as $\vec{\mu}=\vec{\mu}_D+\vec{\mu}_a$ so that $\vec{\mu}_D$ and $\vec{\mu}_a$ are the Dirac and the anomalous magnetic moments, respectively. the anomalous magnetic moment μ_a one has [7]

$$\mu_a=2j\Omega_k\int r^2dr[G_{nk}^2/(2\ell_k+1)+F_{nk}^2/(2\ell_{-k}+1)], \quad (10)$$

where $\Omega_k=1(-1)$, $\ell_k=-k-1(k)$ for $k<0(k>0)$.

The Dirac magnetic moment μ_D one has [6]

$$\mu_D=-\frac{j}{2}\int r^2\,dr[G_{nk}^2-F_{nk}^2-\Omega_kG_{nk}^2/(2\ell_k+1)$$
$$-\Omega_kF_{nk}^2/(2\ell_{-k}+1)]B_f(r) \quad (11)$$

where the reduction factor $B_f(r)$ is defined as

$$B_f(r)=\frac{g_{\Lambda v}}{g_{Nv}}\times\frac{M_N}{M_\Lambda}[1+\frac{\sqrt{(3\pi^2\rho_N/2)^{2/3}+(M_N+V_{so}(r))^2}}{\lambda_{Nv}\rho_N}]^{-1} \quad (12)$$

where M_Λ, M_N and ρ_N are the mass of Λ, nucleon and the density distribution of the nuclear core, respectively, $g_{\Lambda v}/g_{Nv}=2/3$ and $\lambda_{Nv}=g_{Nv}^2/m_v^2$ in which $g_{Nv}=13$ and $m_v=784MeV$ [7].

Results and discussion

In Table 1, our results are compared with the ones presented in Ref. [7]. It should be noted that in Ref. [7] the effects of core polarization and tensor coupling on the magnetic moments are studied through the Dirac equation in the presence of tensor potential. It is shown that the inclusion of a tensor coupling suppresses the effect of core polarization on magnetic moments. Although, since the hyperon wave functions are not sensitive to the tensor potential, then the magnetic moments with or without tensor potential are almost the same. Therefore, the small differences between both results in Table 1 are a direct consequence of the coupling chosen for the lambda hyperon. in Table 2 we listed our theoretical results for the total magnetic moments of ground and excited Λ-hypernuclei, i.e. lambda hyperons in the states $1s_{1/2}$, $1p_{3/2}$ and $1p_{1/2}$. They are also compared with the values obtained by Ref [7]. As is seen there are good agreements between both results, however, the original of differences arises from tensor coupling effect which is completely ignored in our work due to the complexity of Dirac equation in the presence of tensor coupling term. Note that, in hypernuclei the tensor coupling is essential to reproduce the weak spin-orbit splitting in Lambda hyperon [8].

Table 1. Dirac magnetic moments of lambda-hypernuclei ${}^{13}_{\Lambda}C$, ${}^{17}_{\Lambda}O$ and ${}^{41}_{\Lambda}Ca$ ($\mu_D(10^{-4}$n.m)

HYPER NUCLEI	$1s_{1/2}$	REF [7]	$1p_{3/2}$	REF [7]	$1p_{1/2}$	REF [7]
${}^{13}_{\Lambda}C$	-390	-395	-329	-322	-224	-215
${}^{17}_{\Lambda}O$	-489	-483	-513	-503	-378	-369
${}^{41}_{\Lambda}Ca$	-711	-701	-1046	-1065	-556	-546

Table 2. Magnetic moments of ground and excited lambda hypernuclei ${}^{13}_{\Lambda}C$, ${}^{17}_{\Lambda}O$ and ${}^{41}_{\Lambda}Ca$ (μ(n.m)).

	$1s_{1/2}$		$1p_{3/2}$		$1p_{1/2}$	
HYPER NUCLEI	This work	Ref [7]	This work	Ref [7]	This work	Ref [7]
${}^{13}_{\Lambda}C$	-0.619	-0.651	-0.636	-0.644	0.179	0.184
${}^{17}_{\Lambda}O$	-0.627	-0.660	-0.647	-0.662	0.157	0.170
${}^{41}_{\Lambda}Ca$	-0.642	-0.682	-0.701	-0.718	0.148	0.153

Conclusions

Among all, the magnetic moment of Λ-hypernuclei is one of the most important observables related to hypernuclear physics In this work, in a relativistic approach we determined the magnetic moments of several lambda hypernuclei, analytically, considering the spin-orbit potential. Using the NU method, a first effort is made to calculate the wave function analytically and the magnetic moments are computed for the ground and excited states of several lambda hypernuclei.

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